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# Electrical Machines

————— **Christine Foyer**



# MAGNETIC CIRCUITS AND MAGNETIC MATERIALS

## INTRODUCTION

The law of conservation of energy states that the energy cannot be related or destroyed but it can be converted from one form to other. An electrical energy does not occur naturally and also cannot be stored. Hence the efforts are made to generate it continuously to meet the large demands. But to generate an electrical energy means to convert some other form of energy into an electrical form, according to law of conservation of energy. A commonly used method to generate an electrical energy is converting mechanical energy into electrical with the help of a rotating device. Such a machine which converts the mechanical energy into an electrical energy is called a generator. The input mechanical energy can be achieved from steam turbines, steam engines or using potential energy of water to run hydraulic turbines. Such a device which inputs a mechanical energy to a

*“The magnetic circuit is the silent conductor that orchestrates the dance of invisible forces.”*

—Nikola Tesla

### LEARNING OBJECTIVES

After studying this chapter, you will be able to:

1. Explain magnetic circuits
2. Discuss about flux linkage, inductance and energy
3. Describe torque
4. Explain permanent magnets

generator is called a prime mover. While converting energy from mechanical to electrical form, some losses take place. The losses are kept to minimum value by properly designing the machine. Practically the efficiencies of large generators are above 90 %.

Magnetic circuit, closed path to which a magnetic field, represented as lines of magnetic flux, is confined. In contrast to an electric circuit through which electric charge flows, nothing actually flows in a magnetic circuit.

In a ring-shaped electromagnet with a small air gap, the magnetic field or flux is almost entirely confined to the metal core and the air gap, which together form the magnetic circuit. In an electric motor, the magnetic field is largely confined to the magnetic pole pieces, the rotor, the air gaps between the rotor and the pole pieces, and the metal frame. Each magnetic field line makes a complete unbroken loop. All the lines together constitute the total flux. If the flux is divided, so that part of it is confined to a portion of the device and part to another, the magnetic circuit is called parallel. If all the flux is confined to a single closed loop, as in a ring-shaped electromagnet, the circuit is called a series magnetic circuit. Magnetic materials are those which are affected by magnetic field and non-magnetic materials are those which are not affected or slightly affected by magnetic field. Maximum types of materials fall under category of non – magnetic material.

### Keyword

**Magnetic flux** is the product of the average magnetic field times the perpendicular area that it penetrates.

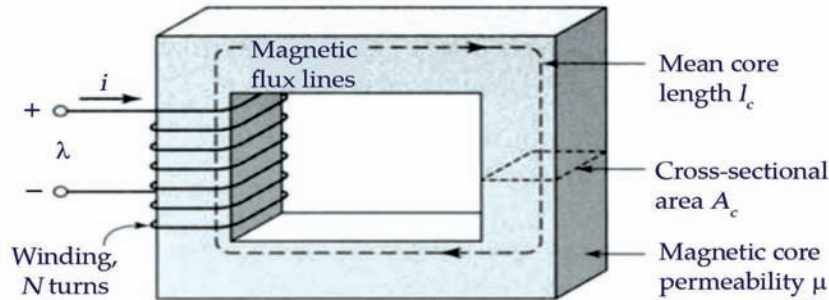
## 1.1 MAGNETIC CIRCUITS

In a magnetic circuit, the magnetic lines of force leaves the north poles passes through the entire circuit and return the starting point. A magnetic circuit usually consist of materials having high permeability such as iron, soft steel etc., these materials offer very small opposition to the flow of **magnetic flux**. Consider a coil of N turns would on an iron core.

Electric machines and transformers have electric circuits and **magnetic circuits** interlinked through the medium of magnetic flux. Electric current flow through the electric circuits, which are made up of windings. On the other hand, magnetic fluxes flow through the magnetic circuits, which are made up of iron cores. The interaction between the currents and the fluxes is the basic of the electromechanical energy conversion process that



takes place in generators and motors. However, in transformers it is more feasible to think about the process in terms of an energy transfer. In transformers, the energy transfer is normally associated with change in voltage and current levels. Thus, magnetic circuits play an essential role.

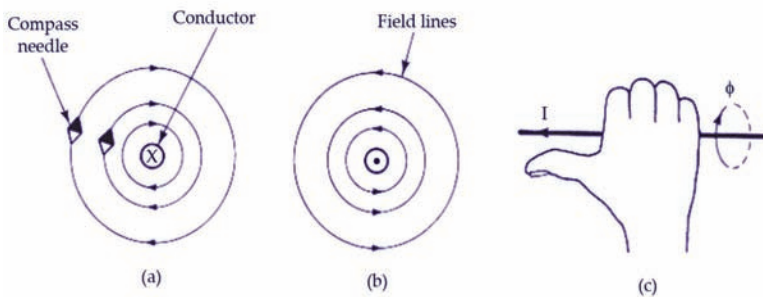


**Keyword**

**Magnetic circuit** is made up of one or more closed loop paths containing a magnetic flux.

**Basics**

The magnetic flux is produced due to the flow of a current in a wire (electric magnet). The direction of the produced magnetic flux is determined by “the right-hand rule” as shown in Figure 1.1.



**Figure 1.1:** The right hand rule for magnetic flux.

The unit for the flux  $\Phi$  is (weber) and the magnetic flux density  $B$  is given as:

$$B = \frac{\phi}{A} \text{ Wb/m}^2 \text{ (Tesla)}$$

The magneto-motive force mmf is the ability of a coil to produce magnetic flux. The mmf unit is Amp-turn and is given

## 4 Electrical Machines

by:  $\text{mmf} = NI$  (AT). The magnetic flux intensity  $H$  is the mmf per unit length along the path of the flux and is given by:  $H = \frac{\text{mmf}}{l}$  (AT/m), where  $l$  is the mean or average path length of the magnetic flux in meters.

The relation between the mmf and the flux is governed by the system reluctance  $\mathfrak{R}$ , such that  $\text{mmf} \mathfrak{R} = \phi$ , where the reluctance is given by  $\mathfrak{R} = \frac{l}{\mu A}$ , where

$l$  = The average length of the magnetic core (m)

$A$  = The cross section area ( $\text{m}^2$ )

$\mu$  = The permeability of the material (AT/m<sup>2</sup>)

The permeability of the material is given by

$$\mu = \mu_0 \mu_r$$

where:  $\mu_0$  is the permeability of air and  $\mu_r$  is the relative permeability.

From the above relationships, we can conclude that:

$$B = \frac{\phi}{A} = \frac{\text{mmf} / \mathfrak{R}}{A} = \frac{(HI) / (l / \mu A)}{A} = \mu H$$

The relation B-H is known as the magnetization characteristics of the material and is broken to three different regions: Linear, knee and saturation as shown in Figure 1.2.

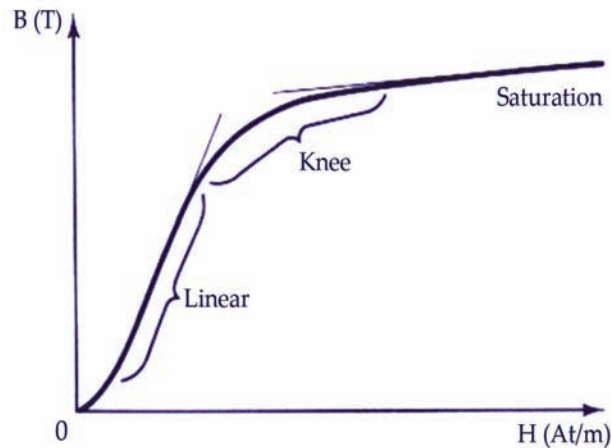


Figure 1.2: Magnetization Curve.

### Analogy between Magnetic and Electric Circuits

ELECTRIC CIRCUIT	MAGNETIC CIRCUIT
$E = \text{EMF}$	$F = \text{MMF}$
$R = \text{Resistance}$	$\mathfrak{R} = \text{Reluctance}$
$I = \text{Current}$	$\phi = \text{Flux}$
$\sigma = \text{Conductivity}$	$\mu = \text{Permeability}$
$E = RI$	$\text{mmf} = \mathfrak{R}\phi$
$R = \frac{l}{\sigma A} = \frac{\rho l}{A}$	$\mathfrak{R} = \frac{l}{\mu A}$
$R_{\text{series}} = R_1 + R_2 + \dots + R_N$	$\mathfrak{R}_{\text{series}} = \mathfrak{R}_1 + \mathfrak{R}_2 + \dots + \mathfrak{R}_N$
$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$	$\frac{1}{\mathfrak{R}_{\text{parallel}}} = \frac{1}{\mathfrak{R}_1} + \frac{1}{\mathfrak{R}_2} + \dots + \frac{1}{\mathfrak{R}_N}$

#### 1.1.1 Magnetic Circuit Analysis

In order to analyze any magnetic circuit, two steps are mandatory as illustrated by Figure 1.3:

- Step #1: Find the electric equivalent circuit that represents the magnetic circuit.
- Step #2: Analyze the electric circuit to solve for the magnetic circuit quantities.

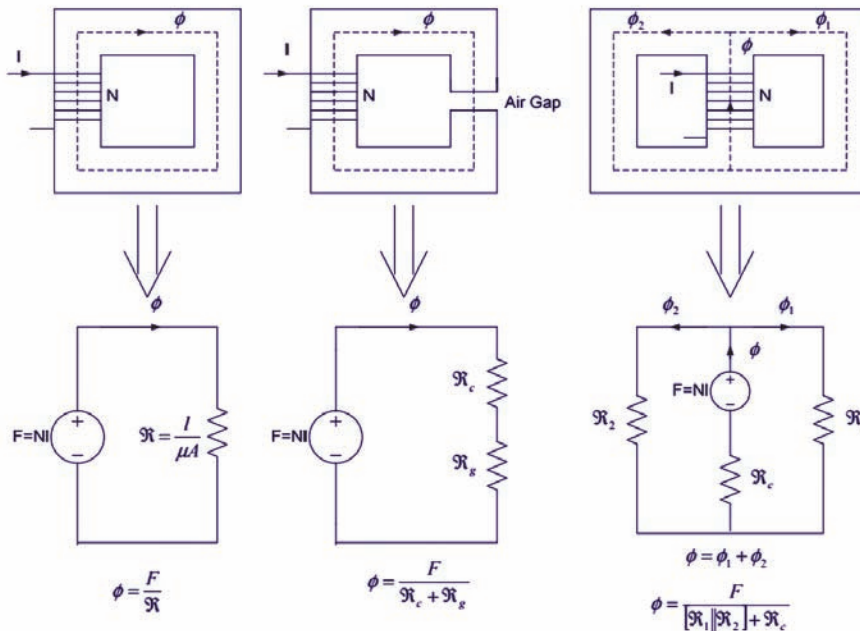


Figure 1.3: Magnetic circuit analysis.

**Example**

The magnetic circuit shown below has the following dimensions:  $A_c = 16 \text{ cm}^2$ ,  $l = 40 \text{ cm}$ ,  $l_g = 0.5 \text{ mm}$  and  $N = 350$  turns. The core is made of a material with the B-H relationship given below. For  $B = 1.0 \text{ T}$  in the core, find:

- The flux  $\phi$  and the total flux linkage  $\lambda$ , where  $\lambda = N \phi$ .
- The required current to set this flux if there is no air gap.
- The required current with the presence of an air gap.

B (Tesla)	H (A.T)
0.6	12.5
0.8	15.0
1.0	20.0
1.2	31.0
1.4	55.0

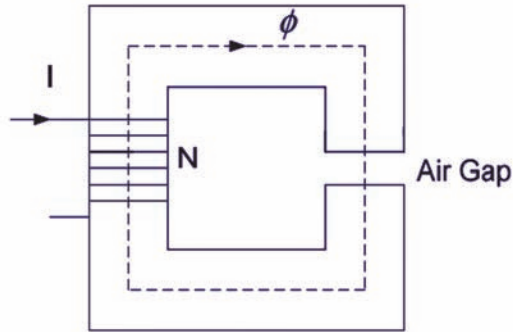


Figure 1.4: Magnetic circuit for the example.

**Solution**

$$\phi = BA_c = 1.0 \times 16 \times 10^{-4} = 1.6 \text{ mWb}$$

a.  $\lambda = N\phi = 350 \times 1.6 \times 10^{-3} = 0.56 \text{ Wb.t}$

b. With no air-gap

$$F = \mathfrak{R}_c \phi = NI$$

$$\therefore I = \frac{\mathfrak{R}_c \phi}{N}$$

$$\mathfrak{R}_c = \frac{l}{\mu_c A_c},$$

$$\mu_c = \frac{B}{H} = \frac{1.0}{20.0} = 0.05$$

$$\mathfrak{R}_c = \frac{40 \times 10^{-2}}{0.05 \times 16 \times 10^{-4}} = 5000 \text{ At/wb}$$

$$\therefore I = \frac{5000 \times 1.6 \times 10^{-3}}{350} = 22.86 \text{ mA}$$

c. With air-gap

$$F = NI = (\mathfrak{R}_c + \mathfrak{R}_g)\phi$$

$$\mathfrak{R}_c = \frac{l_c - l_g}{\mu_c A_c} \cong 5000,$$

$$\mathfrak{R}_g = \frac{l_g}{\mu_g A_g} = \frac{0.5 \times 10^{-3}}{(4\pi \times 10^{-7}) \times 16 \times 10^{-4}} = 248,679.6$$

$$I = \frac{(\mathfrak{R}_c + \mathfrak{R}_g)\phi}{N} = 1.16 \text{ A}$$

In this example, it is clear that the current needed to set the same flux in case of magnetic circuits with air gap compared to those circuits without air-gap is much higher.

### Fringing Effect

The fringing effect results from the presence of the air gap in the magnetic circuit. The main consequence of the fringing effect is to make the magnetic flux density of the air gap different from the flux density of the core due to the path of the flux.

$$\phi_c = \phi_g,$$

but

$$B_c \neq B_g$$

Sometimes air gaps are introduced in the magnetic circuits to linearize the B-H curve. For a given current, the flux density will be smaller due to the air gap presence and saturation is not reached.

### Leakage Flux

In order to set up a specific flux  $\phi_c$  in the core, more MMF is needed due to the flux leakage phenomenon. *Note:* In most

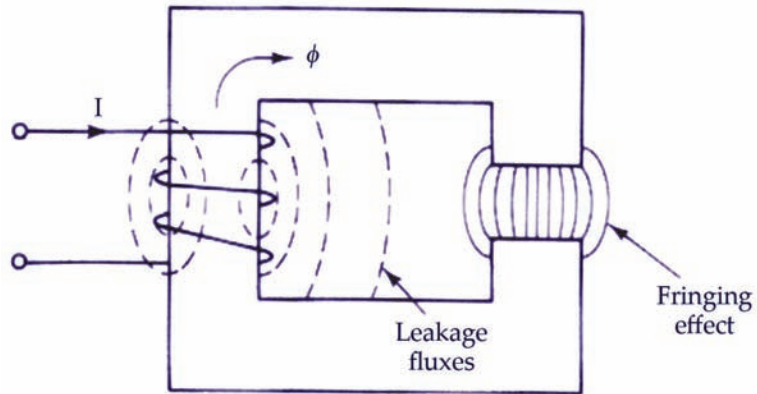
#### Keyword

##### Leakage flux

alternately stores and discharges magnetic energy with each electrical cycle acting as an inductor in series with each of the primary and secondary circuits.



magnetic circuit analysis, both **leakage flux** and fringing effect are neglected.



**Figure 1.5:** Fringing effect and leakage flux.

For a solenoid, suppose the core has a radius of 0.01 m = 1cm and length of 0.2 m (20 cm). Determine the no. of turns required for a current of 1A to produce a magnetic flux density of 0.1 T in the core when the core material is a) air b) iron having relative permeability of 1200.

We know that  $N = Bl / \mu_i$

a) For an air core,  $\mu = \mu_0 = 4\pi \times 10^{-7} \text{ Wb/A-m}$



$$N = \frac{(0.1)(0.2)}{(4\pi \times 10^{-7})} = 15,900 \text{ turns}$$

a) For an air core,  $\mu = \mu_r \mu_0 = 4\pi \times 10^{-7} \text{ Wb/A-m}$

$$N = \frac{(0.1)(0.2)}{(1200)(4\pi \times 10^{-7})} = 13.3 \text{ turns}$$

Exercise: For a solenoid , suppose that the core has a radius of 0.01 m and length of 0.2 m. Find the magnetic flux in the core with 100 turns when the current through the coil is 0.1 A and the core material is a) air b) iron core having relative permeability of 1500.

Answer: a)  $1.97 \times 10^{-7} \text{ Wb}$  , b)  $2.96 \times 10^{-4} \text{ Wb}$



## 1.1.2 Magnetic Material

Magnetic materials are classified based on the property called permeability as

- Dia Magnetic Materials
- Para Magnetic Materials
- Ferro Magnetic Materials

### **Dia Magnetic Materials**

The materials whose permeability is below unity are called Dia magnetic materials. They are repelled by magnet.

*Example:* Lead, gold, copper, glass, mercury

### **Para Magnetic Materials**

The materials with permeability above unity are called Para magnetic materials. The force of attraction by a magnet towards these materials is low.

*Example:* Copper Sulphate, Oxygen, Platinum, Aluminum.

### **Ferro Magnetic Materials**

The materials with permeability thousands of times more than that of paramagnetic materials are called Ferro magnetic materials. They are very much attracted by the magnet.

*Example:* Iron, Cobalt, Nickel.

### **Permanent Magnet**

Permanent magnet means, the magnetic materials which will retain the magnetic property at all times permanently. This type of magnets is manufactured by aluminum, nickel, iron, cobalt steel (ALNICO).

To make a permanent magnet a coil is wound over a magnetic material and DC supply is passed through the coil.

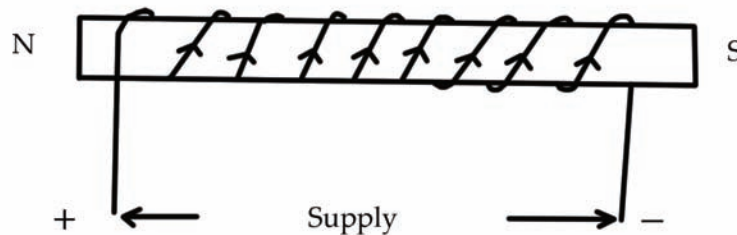
#### Keyword

**Electromagnet** is a type of magnet in which the magnetic field is produced by an electric current.



### Electro Magnet

Insulated wire wound on a bobbin in many turns and layers in which current is flowing and a soft iron piece placed in the bobbin is called **electromagnet**.



This is used in all electrical machines, transformers, electric bells. It is also used in a machine used by doctors to pull out iron filing from eyes, etc.

### Magnetic Effect by Electric Current

If current passes through a conductor magnetic field is set up around the conductor. The quantity of the magnetic field is proportion to the current. The direction of the magnetic field is found by right hand rule or max well's corkscrew rule. Magnetic Flux The magnetic flux in a magnetic circuit is equal to the total number of lines existing on the cross-section of the magnetic core at right angle to the direction of the flux.

#### 1.1.3 Laws Governing Magnetic Circuits

- **Magnetic Flux:** The magnetic lines of force produced by a magnet is called magnetic flux. It is denoted by  $\phi$  and its unit is Weber.
- **Magnetic Field Strength:** This is also known as field intensity, magnetic intensity or magnetic field, and is represented by the letter H. Its unit is ampere turns per meter.

Magnetic field strength is one of two ways that the intensity of a magnetic field can be expressed. Technically, a distinction is made between magnetic field strength H, measured in amperes per meter (A/m), and magnetic flux density B, measured in Newton-meters per ampere (Nm/A), also called teslas (T).

The magnetic field can be visualized as magnetic field lines. The field strength corresponds to the density of the field lines. The total number of magnetic field lines penetrating an area is called the magnetic flux. The unit of the magnetic flux is the tesla meter squared ( $T\ m^2$ , also called the weber and symbolized Wb). The older units for the magnetic flux, the maxwell (equivalent to  $10^{-8}$  Wb), and for magnetic flux density, the gauss (equivalent to  $10^{-4}$  T), are obsolete and seldom seen today.

**Flux Density:** The total number of lines of force per square meter of the cross-sectional area of the magnetic core is called flux density, and is represented by the symbol  $B$ . Its SI unit (in the MKS system) is tesla (weber per meter square).

Magnetic flux density diminishes with increasing distance from a straight current-carrying wire or a straight line connecting a pair of magnetic poles around which the magnetic field is stable. At a given location in the vicinity of a current-carrying wire, the magnetic flux density is directly proportional to the current in amperes. If a ferromagnetic object such as a piece of iron is brought into a magnetic field, the “magnetic force” exerted on that object is directly proportional to the gradient of the **magnetic field** strength where the object is located.

**Magneto-Motive Force:** The amount of flux density setup in the core is dependent upon five factors - the current, number of turns, material of the magnetic core, length of core and the cross-sectional area of the core. More current and the more turns of wire we use, the greater will be the magnetizing effect. We call this product of the turns and current the magneto motive force (mmf), similar to the electromotive force (ernf).

$$\text{MMF} = NI \text{ ampere} - \text{turns}$$

Where mmf is the magneto motive force in ampere turns

$N$  is the number of turns,  $A$ .

**Magnetic Reluctance:** In the magnetic circuit there is something analogous to electrical resistance, and is called reluctance, (symbol  $S$ ). The total flux is inversely proportional to the reluctance and so if we denote mmf by ampere turns.

**Residual Magnetism:** It is the magnetism which remains in a material when the effective magnetizing force has been reduced to zero.

**Magnetic Saturation:** The limit beyond which the strength of a magnet cannot be increased is called magnetic saturation.

**End Rule:** According to this rule the current direction when looked from one end of the coil is in clock wise direction then that end is South Pole. If the current direction is in anti-clock wise direction then that end is North Pole.

### Keyword

**Magnetic field** is the magnetic effect of electric currents and magnetic materials.



**Len's Law:** When an emf is induced in a circuit electromagnetically the current set up always opposes the motion or change in current which produces it.

**Electromagnetic Induction:** Electromagnetic induction means the electricity induced by the magnetic field Faraday's Laws of Electro Magnetic Induction

There are two laws of Faraday's laws of electromagnetic induction.

They are,

1. First Law
2. Second Law

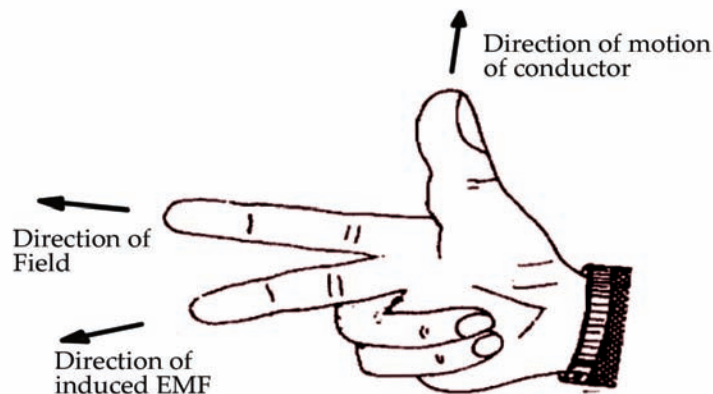
*First Law*

Whenever a conductor cuts the magnetic flux lines an emf is induced in the conductor.

*Second Law*

The magnitude of the induced emf is equal to the rate of change of flux-linkages.

**Fleming's Right Hand Rule:** This rule is used to find out the direction of dynamically induced emf. According to the rule hold out the right hand with the Index finger middle finger and thumb at the right angles to each other's. If the index finger represents the direction of the lines of flux, the thumb points in the direction of motion then middle finger points in the direction of induced current.



## 1.2 FLUX LINKAGE, INDUCTANCE AND ENERGY

Flux linkage is simply the flux density passing through the loop multiplied by its surface area, i.e. . If several turns of the wire are made, this becomes where  $N$  is the number of turns.

When flux is changing with time and relative motion between the coils flux exist between both the coils and conductors and emf induces in both coil and the total induced emf  $e$  is given as:

$$e = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} - \int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

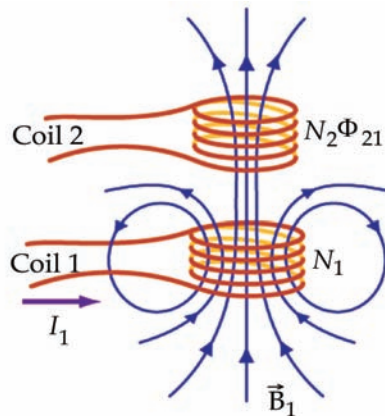
### 1.2.1 Inductance and Energy

A coil wound on a magnetic core, is used frequently used in electric circuits. The coil may be represented by an ideal circuit element called inductance which is defined as the flux linkage of the coil per ampere of its circuit.

$$\begin{aligned} \text{Flux linkage } \lambda &= N\Phi \\ \text{Inductance } L &= \frac{\lambda}{i} \\ L &= \frac{N\Phi}{i} = \frac{NBA}{i} = \frac{N\mu HA}{i} \\ &= \frac{N\mu HA}{HI/N} = \frac{N^2}{l/\mu A} \end{aligned}$$

#### Mutual Inductance

Suppose two coils are placed near each other, as shown in Figure 1.6.



**Figure 1.6:** Changing current in coil 1 produces changing magnetic flux in coil 2.

The first coil has  $N_1$  turns and carries a current  $I_1$  which gives rise to a magnetic field  $\vec{B}_1$ . Since the two coils are close

**Did You Know?**

The magnetic domain theory of how ferromagnetic cores work was first proposed in 1906 by French physicist Pierre-Ernest Weiss, and the detailed modern quantum mechanical theory of ferromagnetism was worked out in the 1920s by Werner Heisenberg, Lev Landau, Felix Bloch and others.

## 14 Electrical Machines

to each other, some of the magnetic field lines through coil 1 will also pass through coil 2. Let  $\Phi_{21}$  denote the magnetic flux through one turn of coil 2 due to  $I_1$ . Now, by varying  $I_1$  with time, there will be an induced emf associated with the changing magnetic flux in the second coil:

$$\varepsilon_{21} = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d}{dt} \iint_{\text{coil 2}} \vec{B}_1 \cdot d\vec{A}_2 \quad (1)$$

The time rate of change of magnetic flux  $\Phi_{21}$  in coil 2 is proportional to the time rate of change of the current in coil 1:

$$N_2 \frac{d\Phi_{21}}{dt} = M_{21} \frac{dI_1}{dt} \quad (2)$$

where the proportionality constant  $M_{21}$  is called the mutual inductance. It can also be written as

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} \quad (3)$$

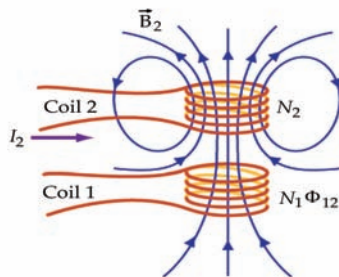
The SI unit for inductance is the henry (H):

$$1 \text{ henry} = 1 \text{ H} = 1 \text{ T} \cdot \text{m}^2 / \text{A} \quad (4)$$

We shall see that the mutual inductance  $M_{21}$  depends only on the geometrical properties of the two coils such as the number of turns and the radii of the two coils. In a similar manner, suppose instead there is a current  $I_2$  in the second coil and it is varying with time (Figure 1.7). Then the induced emf in coil 1 becomes

$$\varepsilon_{12} = -N_1 \frac{d\Phi_{12}}{dt} = -\frac{d}{dt} \iint_{\text{coil 1}} \vec{B}_2 \cdot d\vec{A}_1 \quad (5)$$

and a current is induced in coil 1.



**Figure 1.7:** Changing current in coil 2 produces changing magnetic flux in coil 1.

This changing flux in coil 1 is proportional to the changing current in coil 2,

$$N_1 \frac{d\Phi_{12}}{dt} = M_{12} \frac{dI_2}{dt} \quad (6)$$

where the proportionality constant  $M_{12}$  is another mutual inductance and can be written as

$$M_{12} = \frac{N_1 \Phi_{12}}{I_2} \quad (7)$$

However, using the reciprocity theorem which combines Ampere's law and the BiotSavart law, one may show that the constants are equal:

$$M_{12} = M_{21} \equiv M \quad (8)$$

### Self-Inductance

Consider again a coil consisting of  $N$  turns and carrying current  $I$  in the counterclockwise direction, as shown in Figure 1.8. If the current is steady, then the magnetic flux through the loop will remain constant. However, suppose the current  $I$  changes with time, then according to Faraday's law, an induced emf will arise to oppose the change. The induced current will flow clockwise if  $dI / dt > 0$ , and counterclockwise if  $dI / dt < 0$ . The property of the loop in which its own magnetic field opposes any change in current is called "self-inductance," and the emf generated is called the self-induced emf or back emf, which we denote as  $\epsilon_L$ . All current-carrying loops exhibit this property. In particular, an inductor is a circuit element (symbol) which has a large self-inductance.

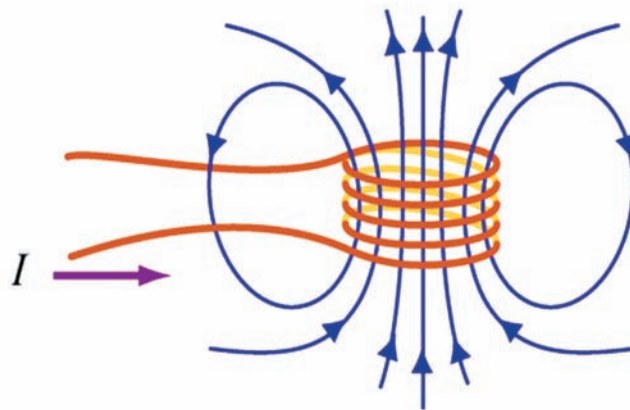


Figure 1.8 Magnetic flux through the current loop.

Mathematically, the self-induced emf can be written as

$$\varepsilon_L = -N \frac{d\Phi_B}{dt} = -N \frac{d}{dt} \iint \vec{B} \cdot d\vec{A} \quad (9)$$

and is related to the self-inductance L by

$$\varepsilon_L = -L \frac{dI}{dt} \quad (10)$$

The two expressions can be combined to yield

$$L = \frac{N\Phi_B}{I} \quad (11)$$

Physically, the inductance L is a measure of an inductor's "resistance" to the change of current; the larger the value of L, the lower the rate of change of current.

## 1.2.2 Statically and Dynamically induced EMF

Induced electro motive forces are of two types. They are,

- Dynamically induced emf.
- Statically induced emf .

### **Statically Induced Emf**

Statically Induced emf is of two types. They are

- Self induced emf
- Mutually induced emf.

### **Self Inductuced emf**

Self-induction is that phenomenon where by a change in the current in a conductor induces an emf in the conductor itself. i.e. when a conductor is given current, flux will be produced, and if the current is changed the flux also changes, as per Faraday's law when there is a change of flux, an emf will be induced. This is called self-induction. The induced emf will be always opposite in direction to the applied emf. The opposing emf thus produced is called the counter emf of self-induction.

Uses of Self induction

- In the fluorescent tubes for starting purpose and to reduce the voltage.
- In regulators, to give reduced voltage to the fans.