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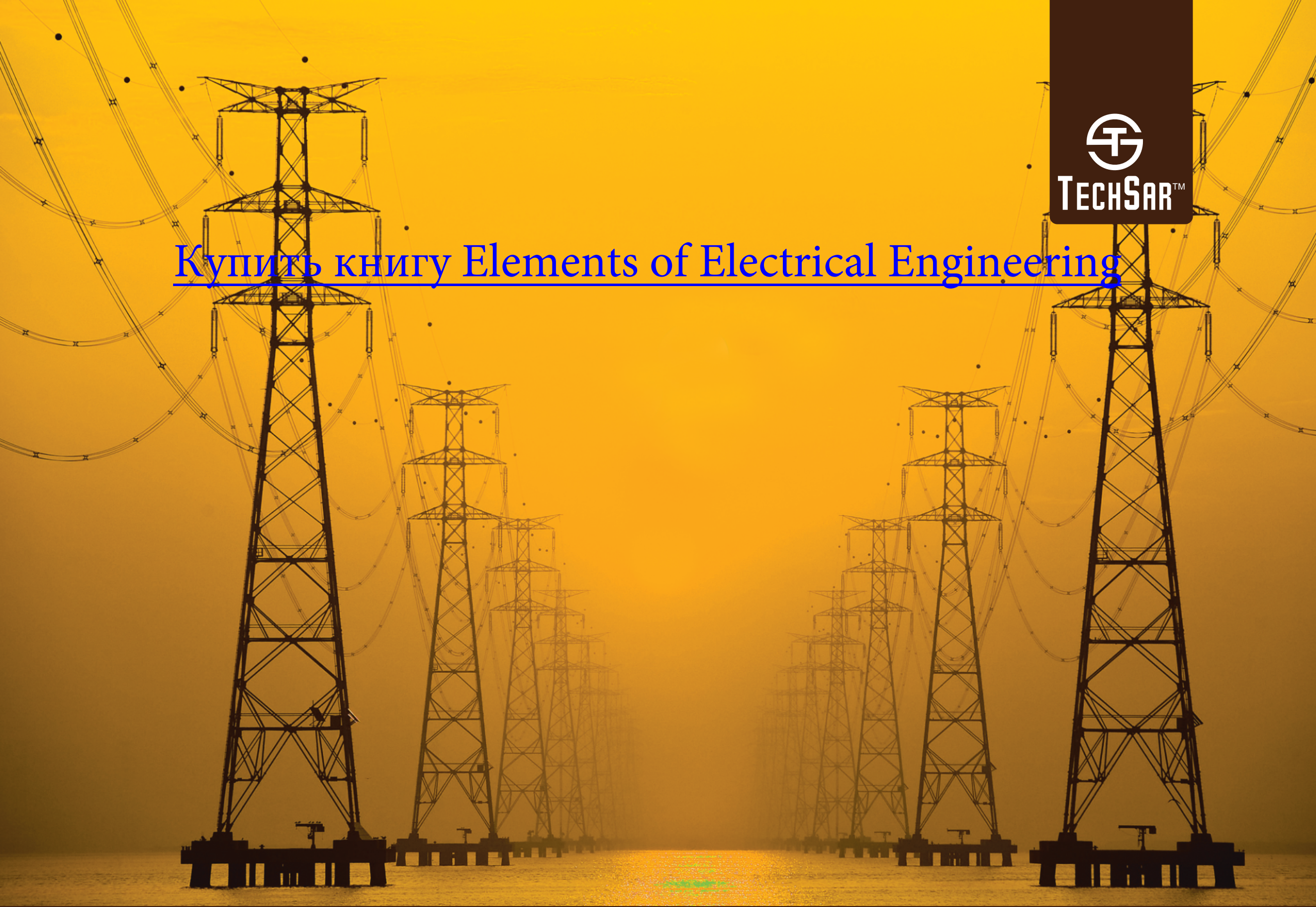

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ELEMENTS OF ELECTRICAL ENGINEERING

PRUTHVIRAJA L

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CHAPTER

DC Circuits

PART–A

INTRODUCTION

The purpose of this chapter is to learn the basics of dc circuits and related problem statements. The use of Ohm's law to the dc circuits and Kirchhoff's laws are well analysed. Even in modern technology, without these fundamental laws we cannot go further to analyze and synthesize the electrical circuits. The Ohm's law is applicable to simple dc circuits and its limitations are also addressed in this chapter. The Kirchhoff's laws are applicable to the circuits where the Ohm's law cannot solve the circuit problem alone. So, both Ohm's law and Kirchhoff's laws together form fundamental key rules in electrical circuit's analysis. We know that, to solve any network problem we need the basic ideas that really help us to get the most skilful information to build the design further. So studying of these rules along with the analysis of circuits with different methods will tell us about the problem conclusion and related issues. The relationship of work, power and energy using Ohm's law is also highlighted in this chapter. So, let us begin with the basic definitions to our real area of interest.

Note: The theory and applications of Kirchhoff's laws discussed in this chapter are only meant for dc circuits, however, Kirchhoff's laws are also applicable for both dc and ac circuits.

1.1 BASIC DEFINITIONS

Charge: We know that materials are made of one or more elements. These elements are composed of entirely from atoms of the same kind. An atom consists of protons, neutrons and electrons. Protons are positively charged particles, whereas electrons are negatively charged and neutrons are electrically neutral. Under normal conditions, an atom is electrically neutral in state. This is due to the total positive charge on protons is equal to the total charge (negative) on electrons even though the electrons move around the nucleus in orbits of a particular distance.

An atom becomes positively charged if it loses electrons and acquires negative charge if it gains excess electrons. So, the charge can be defined as "the quantity of electricity having either positive or negative sign". The unit of charge is Coulomb (C).

Electric Current and its Unit Unless there exists an external electrical effort, the electrons cannot drift in one particular direction. An external electrical force, (or effort) required to drift the electrons in one particular direction in a conducting material, is called "electromotive force (emf). It is measured in volts. So under the influence of an external force, all the electrons move in one particular direction. Electrons flow from negative to positive of the cell, externally through the conductor where it is connected. The electrons flow is always from negative to positive while the movement of current is always assumed from positive to negative. This kind of assumption for current direction is called conventional current direction; it is useful in analysing the electrical circuits. Now, let us define electric current: "electric current is the rate at which the positive charge flows in an electrical circuit or a conductor", i.e., $I = \frac{dq}{dt}$ ampere.

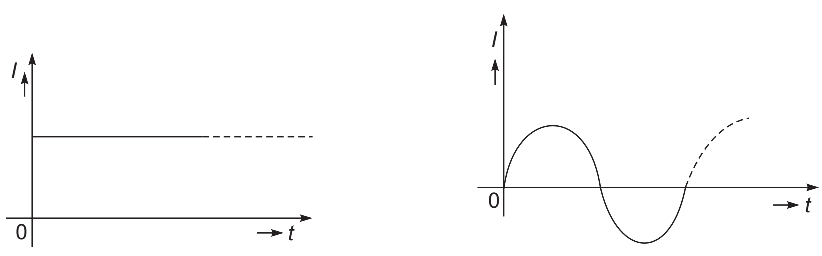
The unit of current is ampere which is equal to coulombs/sec.

1 ampere (A) can be defined as, "the flow of 1C of charge in a conductor at any given point in one second".

Types of Electric Current

DC Current: If the magnitude of the unidirectional current is constant with respect to time, then it is called dc current (Fig. 1.1(a)).

AC Current: If the magnitude of the unidirectional current varies with respect to time and keeps on reversing its direction alternately, then it is called ac current (Fig. 1.1(b)).



(a) (b)

Fig. 1.1 (a) dc current (b) Sinusoidal ac current

Resistivity and Resistance: The property of the conducting material (or) electric circuit which opposes the flow of electric charge (or current) through it is called its resistance. It is denoted by R. The unit of resistance is ohm (Ω).

The resistance of any conducting material is directly proportional to its length (l) and inversely proportional to its area of cross section (a),

i.e.,
$$R \propto \frac{l}{a} \quad \text{or} \quad R = \rho\frac{l}{a}$$

where, ρ is called constant of proportionality referred as *resistivity* of the material, which indicates the effect of nature of the conducting material. It is also called specific resistance of the material. Its unit is ohm metre (Ω-m).

Figure 1.2 shows the details of resistance and resistivity.

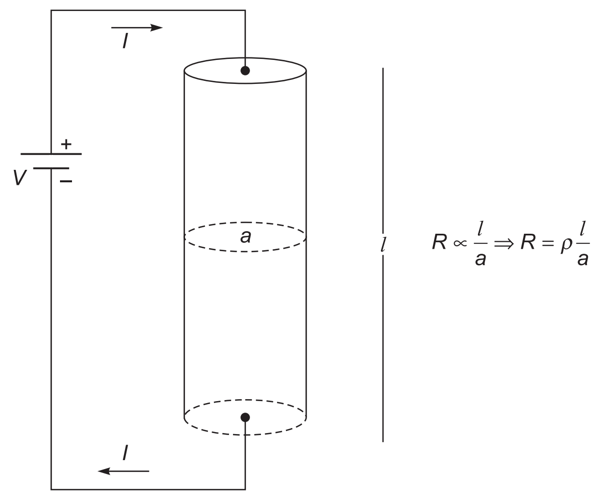


Fig. 1.2 Current flowing is opposed by the resistance

Obsolute Potential: It is defined as the amount of work to be done to bring an unit positive charge from infinity to that point. Electrical potential = $\frac{\text{work done}}{\text{charge}} = \frac{W}{Q}$ = volts V

Potential Difference or Voltage: The difference between the absolute potentials at any two given points in an electric circuit is known as potential difference (p.d.) also called a voltage between the two points. Its unit is volt, V.

For the flow of electrons or current to flow in an electric circuit, there must exist a potential difference between the two points. If the potential difference is zero, it indicates that 'no current' is flowing in that point or the current flow is zero.

We know that, electrons move from negative terminal to positive terminal externally through the circuit and at the same time positive electrons move from positive terminal towards the negative terminal. Therefore, if the charge flow is from positive terminal to negative terminal (or from higher potential to lower potential) then it is called voltage drop. Voltage rise is the reverse of this case. Figure 1.3 illustrates this phenomenon.

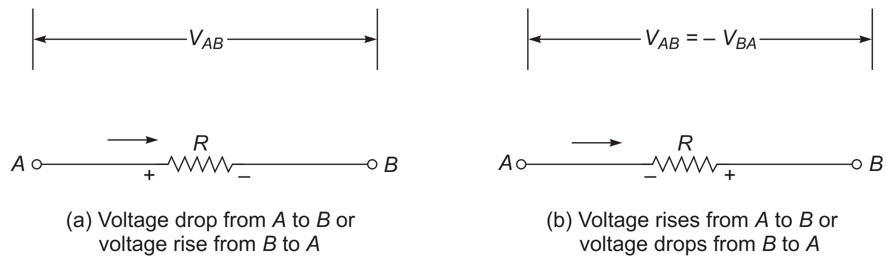


Fig. 1.3 Potential difference between two points

Therefore, the voltage rise in one direction is equal to the voltage drop in the opposite direction. But the sources and batteries are always considered voltage rise devices, through which the current leaves the positive terminal and flows through the devices which are connected to it and makes voltage drop across each individual device. Therefore, there should exist a (+) and (–) sign pair to represent voltage or potential difference which tell us whether there is a voltage rise or voltage drop.

Electric Field in a Conductor: It is defined as the ratio of the potential difference (or voltage) to the length of the conductor terminals.

$$E = \frac{V}{L}$$

where L is the length of the conductor.

The unit is volt/metre (V/m). Because of this electric field only, the current flows in a conductor.

Electromotive Force (emf): Electromotive force is the voltage of the source, such as current source or voltage source or battery, when nothing is connected to it. Electromotive force is not a force but it is called force, because it forces the current to flow in a circuit; it is measured in volts (V).

Terminal Voltage: It is the voltage across the terminals of any source. If the current flows from the terminal voltage of the source to some other element in a circuit, then this terminal voltage is slightly becomes less than the emf because there exists a small voltage drop in its internal resistance.

Sources of Electrical Energy

We know that energy can neither be created nor destroyed from the law of conservation of energy. Yes, but when something is delivering energy means, there must exist something to absorb it. Hence, sources producing energy are called active elements, whereas the devices which are connected to an electric circuit (e.g., resistor, capacitor, inductor, transformer) which absorb electrical energy produced by the active elements are called passive elements. There exist two types of sources:

(i) Voltage source
(ii) Current source

If the sources are functions of voltage or current of other section (or branch) of the system, then they are called dependent sources, otherwise independent sources. Hence, sources may be either dependent or independent.

Independent sources: An ideal voltage source and an ideal current source are independent energy sources. We shall now discuss them in brief.

An ideal voltage source: An ideal voltage source is one whose terminal voltage is independent of the current flowing through its terminals or in other words, the terminal voltage at any time is independent of the output current.

That is, the terminal voltage is independent of the load connected to it. For any value of the load resistance starting from zero to infinity, the terminal voltage remains constant. An ideal voltage source, its characteristics and symbols are shown in Fig. 1.4.

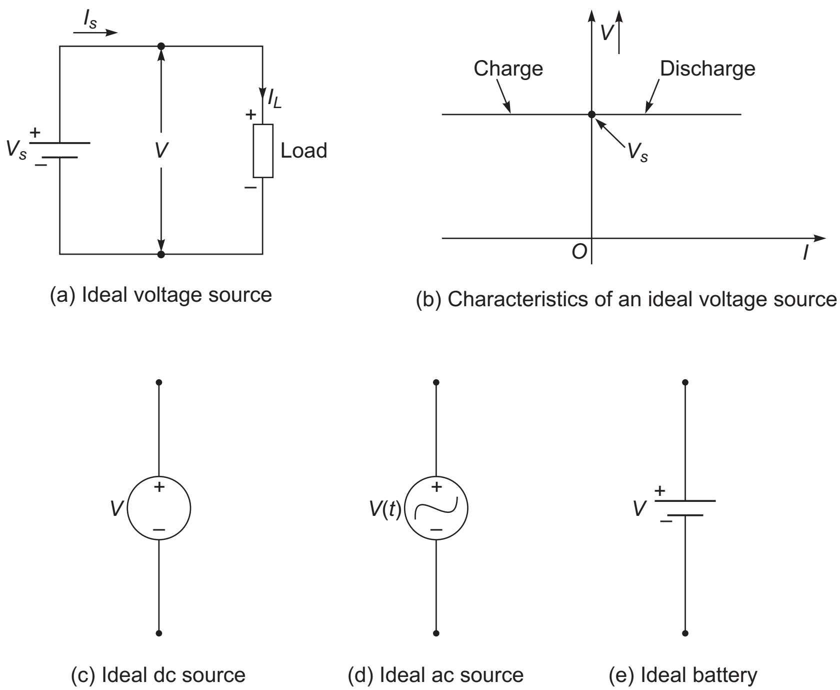


Fig. 1.4 Ideal voltage sources and their symbols

The source determines its voltage. The current flowing through the load terminals is determined by the magnitude of the load and its type.

The current flow from the source is from –ve voltage terminal to the +ve voltage terminal. This means that the source is delivering energy to the load. This current is absorbed by the load, hence the terminals across the load are shown in Fig. 1.4(a).

An Ideal Current Source: Like an ideal voltage source, an ideal current source is the one whose current value through the source is independent of the voltage across its terminals for any type of load (or network), except open circuit, connected to the same terminals.

Here, the magnitude of the current is determined by the source, whereas the voltage across the terminals of the source is determined by the load.

An ideal voltage source, its characteristics and its symbols are shown in Fig. 1.5.

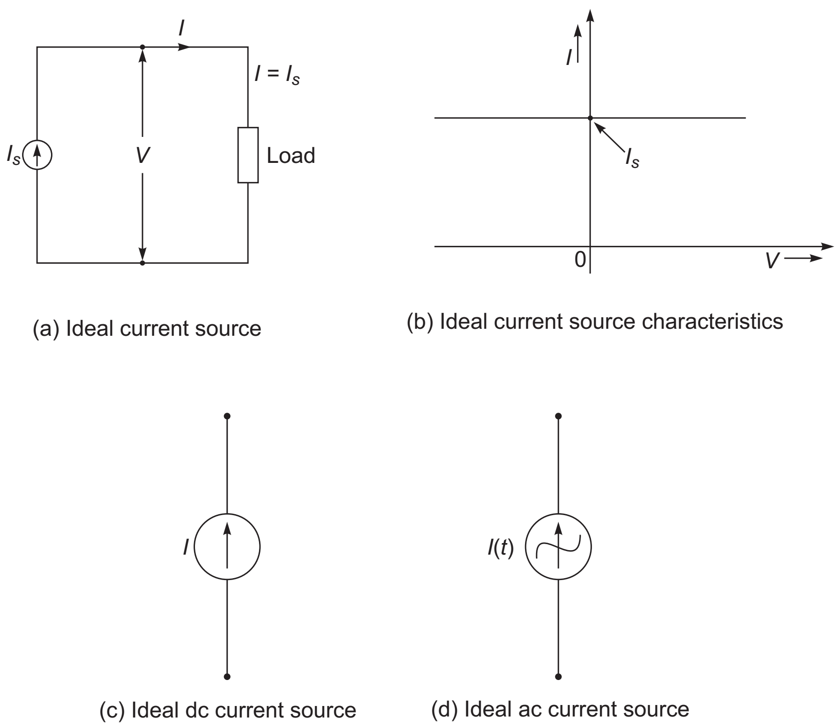


Fig. 1.5 Ideal current sources and their symbols

Practical Voltage Source (or Non-ideal Voltage Source): In general, the terminal voltage of the voltage source no longer remains constant as the load increases. This is because of the reason that each source has its own capacity to do an useful work at its rated load value and beyond this the voltage (or current) does not remain unchanged. To account for this fact, a practical voltage source is developed as an ideal voltage source in series with a small resistance called "internal resistance" or "source resistance". Then the voltage of this ideal source is called electromotive force (emf) of the practical voltage source considered. There is no physical existence of the source resistance R_{se}. It is merely introduced to account for the non-ideality of the practical voltage source and its characteristics. This is shown in Fig. 1.6(a) and Fig. 1.6(b).

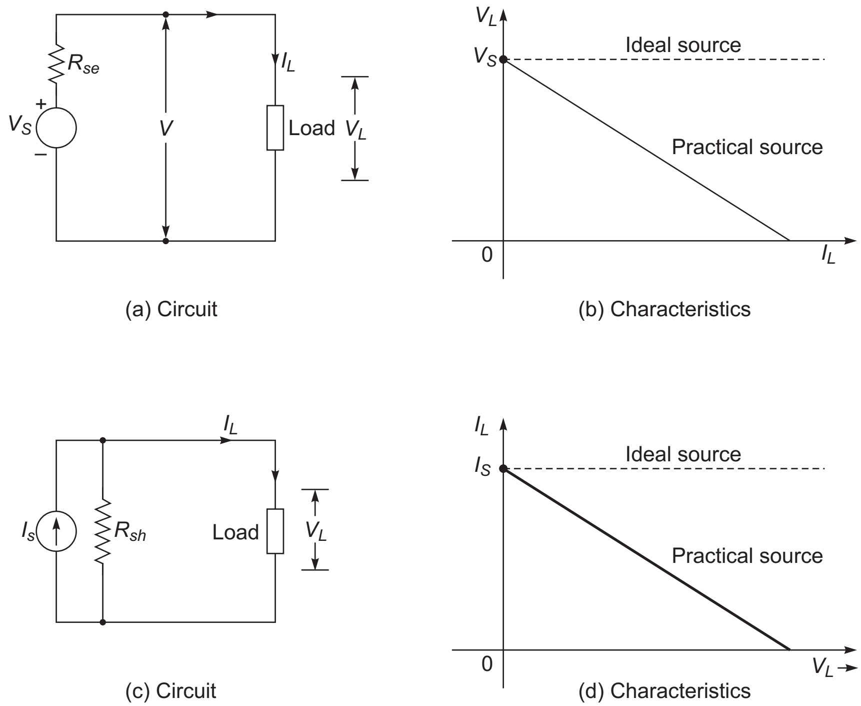


Fig. 1.6 (a) Practical voltage source (c) Practical current source

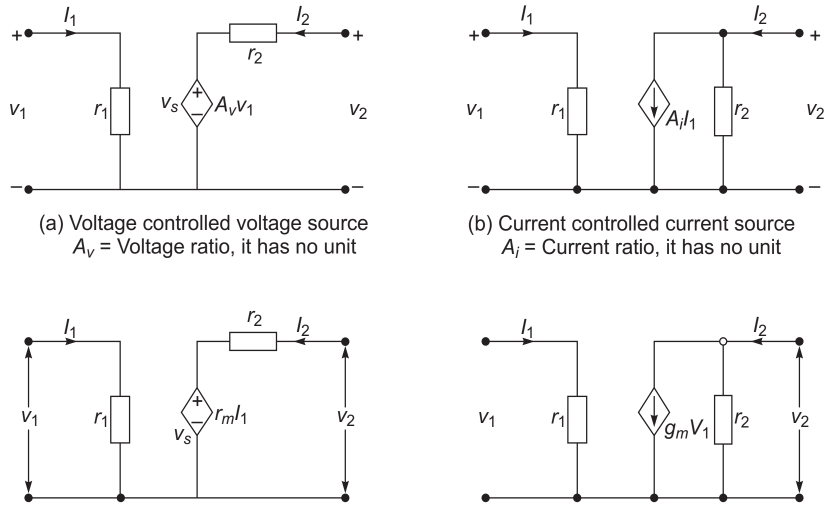
Practical Current Source (or Non-ideal Current Source): A similar explanation can be given for the practical current source also.

Hence, a practical current source is modelled as an ideal current source in parallel with a resistance called internal resistance. The practical current source is the dual of practical voltage source because of source transformation. The practical current source and its characteristics are shown in Figs. 1.6(c) and (d).

Dependent sources: If the terminal voltage (or current through) is a function of the voltage or current elsewhere in the circuit, then the source is called dependent source. Such dependent sources are also called controlled sources. Such sources are schematically represented by the diamond shape.

The output voltage of an op-amp is proportional to the input voltage. So it is an example of voltage controlled voltage source.

In an alternator, the induced emf on the armature coil is proportional to the field current. So it is an example for current controlled voltage source. A transistor is an example of current controlled current source. The transistors can also function as voltage controlled current sources. The various types of dependent sources and their schematic representations are shown in Fig. 1.7.



(a) Voltage controlled voltage source
A_v = Voltage ratio, it has no unit

(b) Current controlled current source
A_i = Current ratio, it has no unit

(c) Current controlled voltage source
r_m = mutual resistance in Ω

(d) Voltage controlled current source
g_m = mutual conductance in siemens (S)

Fig. 1.7 Dependent or controlled sources

1.2 OHM'S LAW

The relationship between the potential difference (V), the current (I) and the resistance (R) of a dc circuit can be obtained from Ohm's law.

Statement:

The Ohm's law states that the current flowing through an electric circuit is directly proportional to the potential difference across that circuit, provided its temperature and other physical parameters remain constant.

That is,

$$I \propto V \quad \text{or} \quad I = \text{constant } (V) \tag{1.1}$$

where the constant of proportionality is called conductance, the reciprocal of the conductance is called resistance, so that Ohm's law can be written as,

$$I = \frac{1}{R} \cdot V \quad \text{or} \quad I = GV \tag{1.2}$$

where $G = \frac{1}{R}$; R is the resistance and G is the conductance.

The SI unit of resistance is ohm (Ω) and that of conductance is siemen (S).

Ohm's Law in graphical form: The Ohm's law in graphical form is obtained when the independent voltage source is connected to a resistor of resistance R. Where the voltage is independent which causes the current to flow which is dependent on the resistor connected across the voltage source. The graph obtained on a linear element resistor is shown in Fig. 1.8, where $\frac{1}{R}$ is called slope of the line and is called conductance.

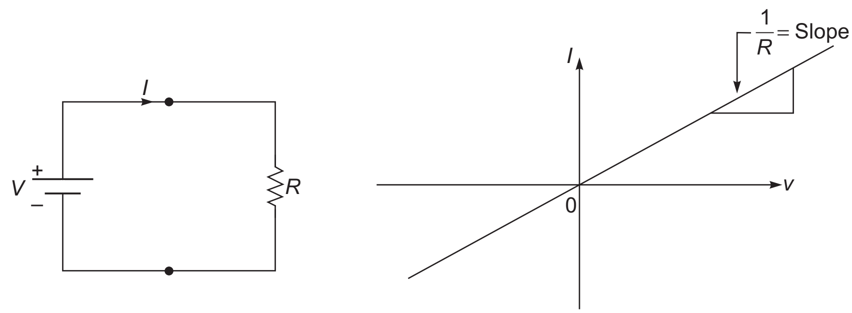


Fig. 1.8 Ohm's law in graphical form

Ohm's law applied to short circuit and open circuit: When there is a short circuit, $R = 0$ (i.e., resistance becomes zero) so the voltage drop across the short-circuited path is $V = IR = I(0) = 0$. But it does not mean that source voltage will fall to zero suddenly. On the other hand, if the circuit is open, then $R = \infty$ which allows infinite opposition for the flow of current in actual meaning so $I = 0$ once again $V \neq 0$. The power going to the resistor is $P = VI$, therefore, no power is required for the short circuit and open circuit. The short circuit and open circuit characteristics are shown in Fig. 1.9.

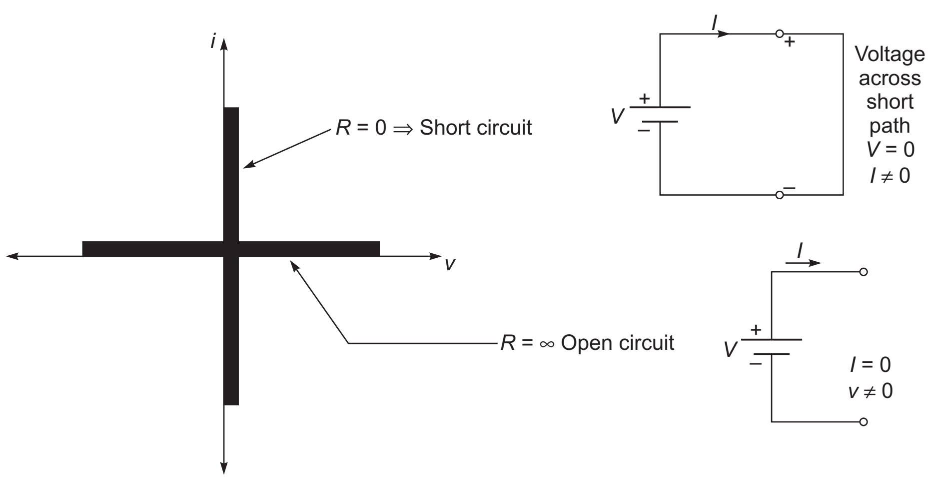


Fig. 1.9 Short circuit and open circuits applied by Ohm's law and their characteristics

1.3 ANALYSIS OF SERIES CIRCUIT EXCITED BY INDEPENDENT VOLTAGE SOURCE

When two or more resistors are connected in series (i.e., end to end) then the resulting circuit is called series circuit. The flow of current remains same (not merely equal) in such circuit.

Figure 1.10(a) shows resistors connected in series and Fig. 1.10(b) shows their equivalent circuit. Look at Fig. 1.10(a), the same current is flowing in all the three resistors, so the voltage drop in each individual resistors will be equal to the total voltage applied (neglecting the source internal resistance).

That is, $V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3$

$\therefore$ $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$

which results in, $$V = I(R_1 + R_2 + R_3) \tag{1.3}$$

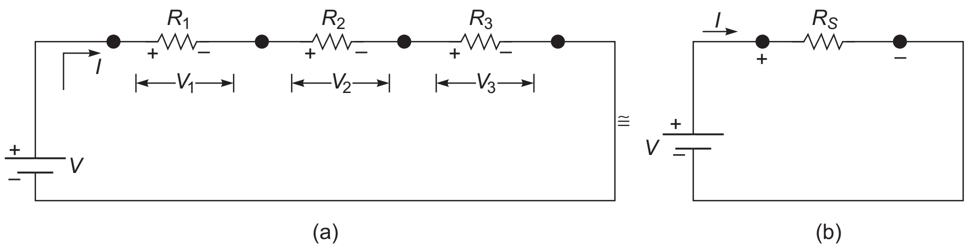


Fig. 1.10 Resistances in series

From Fig. 1.10(b), we can write

$$V = IR_{eq} = IR_s \tag{1.4}$$

From Eqs. (1.3) and (1.4), it can be shown that

$$R_{eq} = R_s = R_1 + R_2 + R_3$$

Therefore, if the n number of resistors are connected in series, then we can write

$$\boxed{R_S = R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots\ R_n = \sum_{i=I}^{n} R_i}$$

If $R_1 = R_2 = R_3 \ldots\ldots\ldots = R_n$

Then $\boxed{R_{sn} = nR}$

1.4 ANALYSIS OF PARALLEL CIRCUIT EXCITED BY INDEPENDENT VOLTAGE SOURCE

If two or more resistors are connected in such a way that the current gets divided into as many number of resistors as connected to the circuit then the resulting circuit is called a parallel circuit.

OR

If the two or more resistors are connected in such a way that the same voltage (merely not equal) exists across them, then the resulting circuit is called a parallel circuit.

Figure 1.11(a) shows the resistors connected in parallel and Fig. 1.11(b) shows their equivalent circuit.

Look at Fig. 1.11(a) the voltage across each resistor is same, but the current in each resistor depends on the value of its individual resistance.

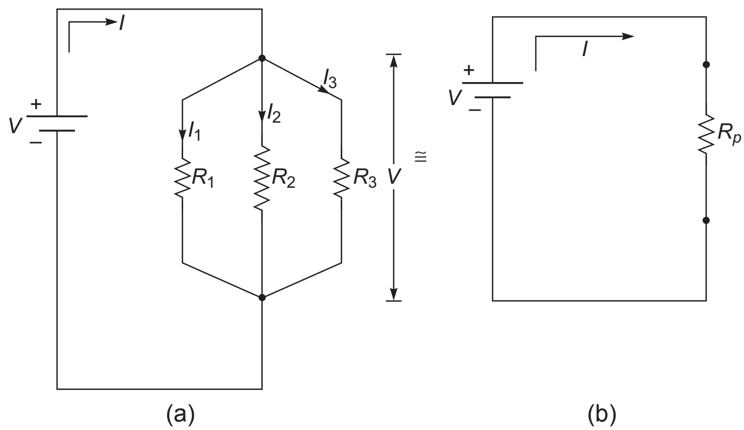


Fig. 1.11 Resistances in parallel

So, $$I_1 = \frac{V}{R_1}; \quad I_2 = \frac{V}{R_2}; \quad I_3 = \frac{V}{R_3}$$

Since the current gets divided and the total current is the sum of all the currents flowing in individual resistors. Therefore, we have

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \tag{1.5}$$

From Fig. 1.11(b), we have

$$I = \frac{V}{R_p} \tag{1.6}$$

From Eqs. (1.5) and (1.6), we can write

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{1.7}$$

In general, if n number of resistors are connected in parallel then the equivalent resistance is given by,

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n} = \sum_{k=1}^{n} \frac{1}{R_k}$$

If $$R_1 = R_2 = R_3 \ldots = R_n$$

Then $$R_P = \frac{R}{n} \quad \text{or} \quad \frac{1}{R_P} = \frac{n}{R}$$

Equation (1.7) can be written as,

$$\frac{1}{R_P} = \frac{R_1R_3 + R_2R_3 + R_1R_2}{R_1R_2R_3}$$

or $$\boxed{R_P = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}} \text{ for 3-resistors in parallel} \tag{1.8}$$

In case of two resistors which are connected in parallel then

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1R_2}$$

or $$\boxed{R_P = \frac{R_1R_2}{R_1 + R_2}} \text{ for two resistors in parallel} \tag{1.9}$$

Voltage and Current Divider Rules: Voltage divider and current divider rules are very important for circuit analysis in electrical engineering.

Let us take a circuit as shown in Fig. 1.12, where two resistors are connected in series.

We know that the same current flows in series circuit

$$\therefore \qquad I = \frac{V}{R_1 + R_2} \quad \text{or} \quad V = I\,(R_1 + R_2)$$

Now, the voltage across R_1 is given by

$$V_1 = IR_1 = R_1\left(\frac{V}{R_1 + R_2}\right) = V\frac{R_1}{R_1 + R_2} \tag{1.10}$$

Similarly,

$$V_2 = IR_2 \quad \text{or} \quad V_2 = V\frac{R_2}{R_1 + R_2} \tag{1.11}$$

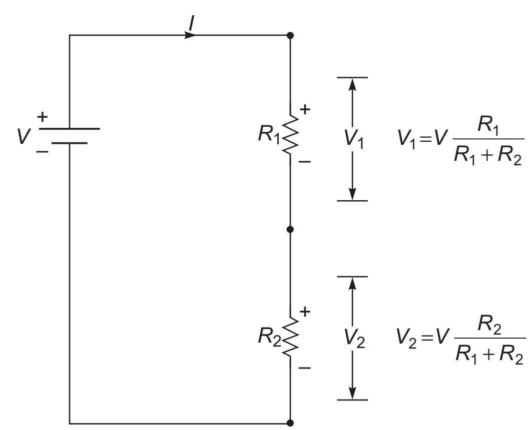


Fig. 1.12 Voltage divider rule

Now, let us take the circuit of Fig. 1.13, where the resistors are connected in parallel.

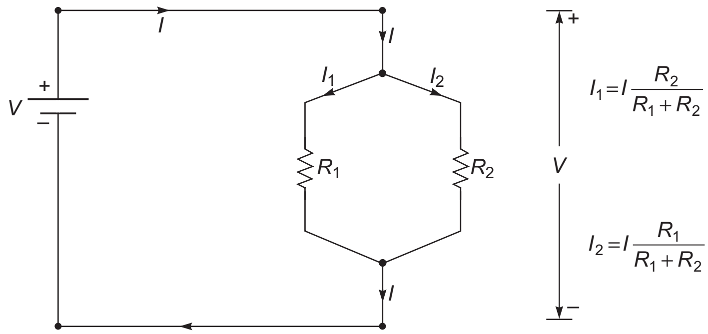


Fig. 1.13 Current divider rule

We know that when the resistors are connected in parallel, current gets divided into as many number of resistors as are connected in the circuit. The total current is the sum of all the individual currents which are flowing in each resistor individually. But voltage across the end terminal remains the same. Therefore, we have

$$I = I_1 + I_2 \quad \text{where} \quad I_1 = \frac{V}{R_1}; \quad I_2 = \frac{V}{R_2}$$

$\therefore$
$$I = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$

or
$$V = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

and $\quad I_1 = \dfrac{V}{R_1}$, substituting V from the above equation, I_1 becomes

$$I_1 = \frac{1}{R_1}\, I\left(\frac{R_1 R_2}{R_1 + R_2}\right) = I\left(\frac{R_2}{R_1 + R_2}\right) \quad \therefore \quad I_1 = I\left[\frac{R_2}{R_1 + R_2}\right] \tag{1.12}$$

Similarly,

$$I_2 = I\left(\frac{R_1}{R_1 + R_2}\right) \tag{1.13}$$

Note: Voltage divider and current divider rules will not follow the same equation. See Eqs. (1.10), (1.11) and (1.12), (1.13).

Thus, we find that when the two resistors are connected in parallel, then the current through any one of the resistors is equal to the total current multiplied by the ratio of the other resistance to the sum of resistances. These voltage and current laws can be applied for more number of resistors as we need. The analogy will be discussed later. Examples on series and parallel (series-parallel) networks are also discussed.